

MODEL FOR FREQUENCY RESPONSE OF A FORCED FLOW,  
HOLLOW, SINGLE TUBE BOILER

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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# MODEL FOR FREQUENCY RESPONSE OF A FORCED FLOW, HOLLOW, SINGLE TUBE BOILER

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## SUMMARY

A simple model is given for the frequency response of boiler inlet pressure to inlet flow. The boiler analyzed is a forced flow, single tube, shell and tube boiler with no inserts. The model yields a simple description of the boiler dynamic characteristics. The characteristics included in the model are boiler pressure drop, time required for the liquid temperature to reach saturation temperature, inertia of the liquid in the subcooled region, and mass storage due to the movement of the boundary between the subcooled and two-phase regions. The model is compared with experimental boiler frequency response data. The boiling fluid was Freon-113, which was heated by counterflowing hot water. The boiler exit quality ranged from 12 to 63 percent. Good agreement between the model and the data was obtained for qualities ranging from 23 to 63 percent.

## INTRODUCTION

Two-phase flow stability problems have been encountered in the development of boiler systems in which liquid is boiled in heated passages. Boiler instabilities can produce degraded system performance and in some cases can lead to system failure. Boiler instabilities, which have been detected by many of the investigators listed in references 1 and 2, are complex in nature and difficult to analyze. An important class of instabilities (manifested by unsteady flow) are those which result from a pressure-flow coupling of the boiler and other loop components (ref. 3). To describe this coupling, the dynamic characteristics of all the loop components must be known. Dynamic techniques have been developed to describe the all-liquid and all-vapor portions of the loop (ref. 4). But due to the complexity of two-phase flow (see ref. 5) and the large number of possible boiler configurations, a general description of boiler dynamic characteristics is not available.

Insight into the dynamic characteristics of boilers can be obtained from the boiler frequency response, that is, the response of one boiler variable to small amplitude sinusoidal perturbations in another variable. This report presents a simple model for the frequency response of boiler inlet pressure to boiler inlet flow. The model includes the effects of boiler pressure drop, the time required for the liquid temperature to reach saturation temperature, inertia of the liquid in the subcooled region, and mass storage due to the movement of the interface between the liquid and two-phase regions of the boiler. The model is compared with the experimental frequency response data reported in reference 6.

The data of reference 6 were obtained by imposing small amplitude sinusoidal perturbations on the pressure and flow at the inlet of a hollow, single tube, shell and tube boiler. A schematic of this boiler is shown in figure 1. Boiler dimensions are given in this figure. The boiling fluid, Freon-113, flowed vertically upward and was heated by counterflowing hot water. The Freon exhausted into a constant pressure plenum tank. Frequency response data were obtained over a frequency range from 0.04 to 4.0 cps (Hz). The data were taken at three nominal values of inlet water temperature: 210°, 230°, and 250° F (372°, 383°, and 394° K). For each inlet water temperature, data were obtained at several Freon flow rates ranging from 175 to 656 pounds per hour (0.022 to 0.083 kG/sec). The Freon inlet temperature was about 70° F (285° K) for all runs. The Freon vapor exit quality ranged from 12 to 63 percent. A detailed description of the experiment is presented in reference 6.

## ANALYSIS

The analysis of the boiler dynamic characteristics is based on the following major assumptions:

- (1) All of the steady-state boiler pressure drop can be lumped at the boiler outlet.
- (2) The axial density distribution can be approximated by assuming constant densities in both the all-liquid and two-phase regions with a discontinuity at the boundary between these regions.
- (3) The density in the two-phase region is much smaller than the liquid density.
- (4) The boundary between the all-liquid and two-phase regions is the point where the bulk liquid temperature reaches saturation.
- (5) Changes in the position of the boundary between the all-liquid and two-phase regions are due only to changes in the flow at the boiler inlet and changes in pressure at the liquid two-phase boundary.

It was also assumed that variations in boiler variables are small amplitude sine waves. Thus equations can be linearized and operator notation can be used for time derivatives.

A schematic of the boiler indicating the major assumptions made in the analysis is shown in figure 2.

Liquid flows into the boiler and is heated until its bulk temperature reaches saturation temperature at  $x = L_{sc}$ . (All symbols are defined in the appendix.) The saturation temperature is determined by the pressure at the liquid two-phase boundary  $P_{L_{sc}}$ . At steady state,  $P_{L_{sc}}$  is assumed to equal the pressure at the boiler inlet  $P_{in}$ , but due to the inertia of the liquid  $P_{L_{sc}}$  will be different from  $P_{in}$  when the liquid is accelerated. When the liquid temperature reaches saturation temperature, it begins to boil and its density is assumed to drop from  $\rho_L$  to  $\rho_{TP}$ . If the flow into the boiler or the pressure at the liquid two-phase boundary changes, the position at which the liquid temperature reaches saturation will move. Neglecting density changes in the two-phase region, the total amount of mass in the boiler will change by an amount  $(\rho_L - \rho_{TP})A_t \Delta L_{sc}$ . The equation for conservation of mass, with  $\rho_{TP} \ll \rho_L$ , is written in small perturbation form as

$$\Delta W_{out} = \Delta W_{in} - \rho_L A_t \frac{d \Delta L_{sc}}{dt} \quad (1)$$

From the assumption that all of the steady-state pressure drop can be lumped at the boiler outlet, and since the steady-state value of  $P_{L_{sc}}$  is assumed to equal the steady-state value of  $P_{in}$ , it follows that

$$\Delta P_{L_{sc}} = \frac{d(P_{in} - P_{out})}{dW_t} \Delta W_{out} = R \Delta W_{out} \quad (2)$$

Combining equations (1) and (2) results in

$$\Delta P_{L_{sc}} = R \left( \Delta W_{in} - \rho_L A_t \frac{d \Delta L_{sc}}{dt} \right) \quad (3)$$

From the assumption that changes in the location of the boundary between the liquid and two-phase regions are functions only of changes in the flow at the boiler inlet and changes in the pressure at the boundary, the following is determined:

$$\Delta L_{sc} = \left( \frac{\Delta L_{sc}}{\Delta W_{in}} \right)_{P_{L_{sc}}} \Delta W_{in} + \left( \frac{\Delta L_{sc}}{\Delta P_{L_{sc}}} \right)_{W_t} \Delta P_{L_{sc}} \quad (4)$$

where  $\left(\Delta L_{sc}/\Delta W_{in}\right)_{P_{L_{sc}}}$  is the transfer function relating changes in the subcooled length to changes in boiler inlet flow and  $\left(\Delta L_{sc}/\Delta P_{L_{sc}}\right)_{W_t}$  is the transfer function which relates changes in subcooled length to pressure changes at the liquid two-phase boundary. The transfer function  $\left(\Delta L_{sc}/\Delta W_{in}\right)_{P_{L_{sc}}}$  was derived from the energy equation for the subcooled region in reference 7. This result is given in the following equation:

$$\left(\frac{\Delta L_{sc}}{\Delta W_{in}}\right)_{P_{L_{sc}}} = \frac{L_{sc}}{W_t} \left( \frac{1 - e^{-j\omega\tau_{sc}}}{j\omega\tau_{sc}} \right) \quad (5)$$

where  $\tau_{sc} = \rho_L A_t L_{sc} / W_t$ .

The derivation in reference 7 assumed that the heat flux in the subcooled region was not a function of position or time. The steady-state heating fluid temperature profiles presented in reference 6 have a slope which is nearly constant in the subcooled region, thus indicating that the heat flux did not vary much with position in the subcooled region. It is also assumed herein that the heat flux is not a function of time. Thus the derivation in reference 7 can be applied to the data of reference 6 and used in this model.

If it is assumed that the change in boundary position due to a pressure change is not a function of frequency, the transfer function  $\left(\Delta L_{sc}/\Delta P_{L_{sc}}\right)_{W_t}$  can be evaluated from steady-state data. Thus  $\left(\Delta L_{sc}/\Delta P_{L_{sc}}\right)_{W_t} = \left(\partial L_{sc}/\partial P_{L_{sc}}\right)_{W_t}$ . Equation (4) can then be written as

$$\Delta L_{sc} = \frac{L_{sc}}{W_t} \left( \frac{1 - e^{-j\omega\tau_{sc}}}{j\omega\tau_{sc}} \right) \Delta W_{in} + \left( \frac{\partial L_{sc}}{\partial P_{L_{sc}}} \right)_{W_t} \Delta P_{L_{sc}} \quad (6)$$

Putting equation (3) in operator notation and combining it with equation (6) give

$$\Delta P_{L_{sc}} = R \left\{ \Delta W_{in} - \rho_L A_t j\omega \left[ \frac{L_{sc}}{W_t} \left( \frac{1 - e^{j\omega\tau_{sc}}}{j\omega\tau_{sc}} \right) \Delta W_{in} + \left( \frac{\partial L_{sc}}{\partial P_{L_{sc}}} \right)_{W_t} \Delta P_{L_{sc}} \right] \right\} \quad (7)$$

Since  $\tau_{sc} = \rho_L A_t L_{sc} / W_t$ ,

$$\Delta P_{L_{sc}} = \frac{Re^{-j\omega\tau_{sc}}}{1 + j\omega R \rho_L A_t \left( \frac{\partial L_{sc}}{\partial P_{L_{sc}}} \right) \frac{W_t}{A_t}} \quad (8)$$

In small perturbation form, the momentum equation for the all-liquid region is

$$\Delta P_{in} = \Delta P_{L_{sc}} + j\omega \frac{L_{sc}}{A_t} \Delta W_{in} \quad (9)$$

Combining equations (8) and (9) yields

$$\frac{\Delta P_{in}}{\Delta W_{in}} = \frac{Re^{-j\omega\tau_{sc}}}{1 + j\omega R \rho_L A_t \left( \frac{\partial L_{sc}}{\partial P_{L_{sc}}} \right) \frac{W_t}{A_t}} + j\omega \frac{L_{sc}}{A_t} \quad (10)$$

Equation (10) gives the dynamic ratio of pressure perturbation at the boiler inlet to flow perturbation at the boiler inlet. Since this ratio was measured in the experiment reported in reference 6, a direct comparison of the model with experimental data can be made.

## EVALUATION OF MODEL PARAMETERS

Since variations in boiler variables were assumed to be small, all of the parameters in equation (10) can be defined in terms of steady-state data. The boiler resistance  $R$  is defined as the slope of the steady-state boiler pressure drop against flow curve. Plots of boiler pressure drop against flow, for the boiler used in reference 6, are shown in figure 3. Each curve in figure 3 is for constant inlet water temperature. Thus for a given inlet water temperature and Freon flow, the boiler resistance can be determined from figure 3.

The subcooled length  $L_{sc}$  is defined as the distance from the boiler inlet at which the Freon temperature reaches saturation temperature. This distance can be determined

from a heat balance between the Freon and the heating fluid (water) and the heating fluid temperature profile. Figure 4 is a plot of subcooled length against flow for various constant inlet water temperatures. The value of  $L_{sc}$  at a particular Freon flow and water temperature was taken from the curve. This value was then used to evaluate  $\tau_{sc}$ ,  $\tau_{sc} = L_{sc}/v_{in}$ , and the liquid inertance  $L_{sc}/A_t$ .

The term  $\left(\frac{\partial L_{sc}}{\partial P_{L_{sc}}}\right)_{W_t}$  is determined from the assumption that changes in the subcooled length are functions of flow and pressure changes only. Thus

$$\frac{dL_{sc}}{dW_t} = \left(\frac{\partial L_{sc}}{\partial W_t}\right)_{P_{L_{sc}}} + \left(\frac{\partial L_{sc}}{\partial P_{L_{sc}}}\right)_{W_t} \frac{dP_{L_{sc}}}{dW_t}$$

Since  $P_{L_{sc}} = P_{in}$  at steady state and  $P_{out}$  is constant,

$$\frac{dP_{L_{sc}}}{dW_t} = \frac{d(P_{in} - P_{out})}{dW_t} = R$$

At steady state,  $\left(\frac{\partial L_{sc}}{\partial W_t}\right)_{P_{L_{sc}}} = \left(\Delta L_{sc}/\Delta W_t\right)_{P_{L_{sc}}, \omega=0}$ . Thus from equation (5),

$$\left(\frac{\partial L_{sc}}{\partial W_t}\right)_{P_{L_{sc}}} = \frac{L_{sc}}{W_t}$$

and

$$\left(\frac{\partial L_{sc}}{\partial P_{L_{sc}}}\right)_{W_t} = \frac{\frac{dL_{sc}}{dW_t} - \frac{L_{sc}}{W_t}}{R}$$

where  $dL_{sc}/dW_t$  is obtained from the curves shown in figure 4. Boiler steady-state conditions and model parameters are listed in table I.



## DISCUSSION OF BOILER MODEL AND COMPARISON WITH DATA

The ratio of boiler inlet pressure perturbation to boiler inlet flow perturbation has been derived herein by solving conservation equations for the boiling process. These equations were based on a simplified picture of the boiling process. The final result is given in equation (10). This ratio of boiler inlet pressure perturbation to boiler inlet flow perturbation is referred to as the boiler inlet impedance. The term  $R$  in equation (10) is referred to as the boiler resistance and is defined as the slope of the steady-state boiler pressure drop against flow curve. Since the steady-state boiler pressure drop was placed at the boiler exit, pressure perturbations due to this pressure drop are produced by flow perturbations out of the boiler. The quantity

$e^{-j\omega\tau_{sc}} \left/ \left[ 1 + j\omega R \rho_L A_t \left( \frac{\partial L_{sc}}{\partial P_{L_{sc}}} \right)_{W_t} \right] \right.$ , which multiplies the boiler resistance, is the

ratio of flow perturbation at the boiler exit to flow perturbation at the boiler inlet. Thus, the first quantity in equation (10) is the ratio of the pressure perturbation caused by exit

flow perturbation to the inlet flow perturbation. The term  $e^{-j\omega\tau_{sc}}$  is the result of mass storage in the boiler due to the dependency of the position of the liquid two-phase boundary on boiler inlet flow. This term indicates that there is a dead time  $\tau_{sc}$  between flow out of the boiler and flow into the boiler. That is, the flow out of the boiler at time  $t$  equals the flow into the boiler at time  $t - \tau_{sc}$ . For sinusoidal flow perturbations, this dead time produces a phase lag between flow perturbations out of the boiler and flow perturbations into the boiler which increases linearly with frequency. The dead time has no effect on the amplitude ratio of the flow perturbations.

The term  $R \rho_L A_t \left( \frac{\partial L_{sc}}{\partial P_{L_{sc}}} \right)_{W_t}$  is due to mass storage as a result of the dependency of the position of the liquid two-phase boundary on pressure. This mass storage produces a decrease in the amplitude ratio of the flow perturbations as frequency increases and produces up to  $90^\circ$  phase lag between flow perturbation out of the boiler and flow perturbation into the boiler. The decrease in amplitude ratio is due to the fact that, as frequency increases, more of the inlet flow perturbation goes into mass storage and less out of the boiler.

The quantity  $j\omega L_{sc}/A_t$  comes from the momentum equation for the liquid region of the boiler. This quantity has very little effect on the boiler impedance for frequencies below 1 cps (Hz) since its magnitude is much less than that of the quantity

$\text{Re}^{-j\omega\tau_{sc}} \left/ \left[ 1 + j\omega R \rho_L A_t \left( \frac{\partial L_{sc}}{\partial P_{L_{sc}}} \right)_{W_t} \right] \right.$ . At higher frequencies, where the magnitudes

of these two quantities are nearly equal, the two quantities can subtract or add to produce

sharp dips or peaks in the magnitude of the boiler impedance.

A direct comparison of the model derived in this report with the data from reference 6 is presented in figure 5. This figure consists of polar plots of the complex ratio of boiler inlet pressure perturbation to boiler inlet flow perturbation. The solid curve was obtained from the model (eq. (10)), and the dashed curve was obtained by drawing a smooth curve through the data of reference 6. Figure 5 indicates that both the model and the data have very similar characteristics over most of the frequency range for these nine runs. Two runs not compared (runs 1 and 2 of ref. 6) had low vapor exit qualities (12 and 13 percent). The 13 percent run was on the negative slope region of the boiler pressure drop against flow curve. The 12 percent run was near this region. Satisfactory agreement between the model and the data was not obtained for these two runs. Thus the model does not appear to work near the negative slope region of boiler pressure drop against flow. Stenning and Veziroglu (ref. 8) also found that a single model would not describe boiler dynamic characteristics in both the positive and negative slope regions of the boiler pressure drop against flow curve.

In addition to the comparison in polar form, the model and the data are compared in plots of amplitude and phase against frequency in figure 6 for several of the runs. The solid curve was obtained from the model. The circles are the actual data points. And the dashed curve is a fit of the lower frequency data. It was found that a curve of the form  $Ke^{-j\omega\tau_1}/(1 + j\omega\tau_2)$  fit the lower frequency data quite well for all but two of the runs (runs 6 and 9) that are compared with the model. The form of the equation used for the data fit is identical to the equation derived in this report (eq. (10)) for cases where the term  $j\omega L_{sc}/A_t$  in the model is negligible. Thus for low frequency, where  $j\omega L_{sc}/A_t$  is small, the model and the data fit should coincide and there should be a one-to-one correspondence between the parameters of the model and those of the data fit. Thus  $R$  should equal  $K$ ,  $\tau_{sc}$  should equal  $\tau_1$ , and  $R\rho_L A_t \left( \partial L_{sc} / \partial P_{L_{sc}} \right)_{W_t}$  should equal  $\tau_2$  if the parameters of the boiler model were evaluated properly. Values of  $\tau_1$  and  $\tau_2$  were obtained for the seven runs which could be fit by the equation presented before. Since the value of  $K$  is determined only by the very low frequency amplitude,  $K$  is compared with  $R$  for all nine runs. Figures 7 to 9 are plots of the parameters in the data fit against the corresponding parameters in the model. Good agreement is obtained for all three parameters compared.

## RÉSUMÉ

The model presented in this report yields a simple expression relating perturbations in boiler inlet pressure to perturbations in boiler inlet flow. The model includes the

effects of boiler pressure drop, time required for the liquid temperature to reach saturation, inertia of the liquid in the subcooled region, and mass storage due to the movement of the boundary between the subcooled and two-phase regions. The model is compared with frequency response data reported in reference 6. The model and the data have very similar characteristics over most of the frequency range for nine of the eleven runs reported in reference 6. In addition, three of the four model parameters are compared with corresponding parameters from a fit of the data of reference 6. Good agreement is obtained.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, December 11, 1967,  
120-27-04-27-22.

## APPENDIX - SYMBOLS

$A_t$	cross-sectional area of boiler tube, $\text{ft}^2$ ( $\text{m}^2$ )	$t$	time, sec
$j$	$\sqrt{-1}$	$v_{\text{in}}$	liquid velocity at boiler inlet, $\text{ft/sec}$ ( $\text{m/sec}$ )
$L_{\text{sc}}$	subcooled length, ft (m)	$\Delta W_{\text{in}}$	perturbation in flow at boiler inlet, slug/sec (kg/sec)
$\Delta L_{\text{sc}}$	perturbation in subcooled length, ft (m)	$\Delta W_{\text{out}}$	perturbation in flow at boiler outlet, slug/sec (kg/sec)
$P_{\text{in}}$	pressure at boiler inlet, $\text{lb/ft}^2$ ( $\text{N/m}^2$ )	$W_t$	boiler flow rate, slug/sec (kg/sec)
$\Delta P_{\text{in}}$	perturbation in boiler inlet pressure, $\text{lb/ft}^2$ ( $\text{N/m}^2$ )	$x$	position, ft (m)
$P_{L_{\text{sc}}}$	pressure at $x = L_{\text{sc}}$ , $\text{lb/ft}^2$ ( $\text{N/m}^2$ )	$\rho_L$	liquid density, slug/ $\text{ft}^3$ ( $\text{kg/m}^3$ )
$\Delta P_{L_{\text{sc}}}$	perturbation in pressure at $x = L_{\text{sc}}$ , $\text{lb/ft}^2$ ( $\text{N/m}^2$ )	$\rho_{\text{TP}}$	two-phase density, slug/ $\text{ft}^3$ ( $\text{kg/m}^3$ )
$P_{\text{out}}$	pressure at boiler outlet, $\text{lb/ft}^2$ ( $\text{N/m}^2$ )	$\tau_{\text{sc}}$	time required for liquid temperature to reach saturation, sec
$R$	boiler resistance, $(\text{lb/ft}^2)/(\text{slug/sec})$ [ $(\text{N/m}^2)/(\text{kg/sec})$ ]	$\omega$	frequency, rad/sec

## REFERENCES

1. Gouse, S. William, Jr.: An Index to the Two-Phase Gas-Liquid Flow Literature. Part 1. Rep. No. DSR-8734-1 (DDC No. AD-411512), May 1963; Part 2. Rep. No. DSR-8734-4 (DDC No. AD-607180), Massachusetts Inst. Tech., Sept. 1964.
2. Balzhiser, R. E.; Clark, J. A.; Colver, C. P.; Hucke, E. E., and Merte, H., Jr.: Literature Survey on Liquid Metal Boiling. Rep. No. 04526-2-F (AFASD-TR-61-594), Michigan University, Dec. 1961.
3. Dorsch, Robert G.: Frequency Response of a Forced-Flow Single-Tube Boiler. Paper presented at the Symposium on Two Phase Flow Dynamics, sponsored by Technological University of Eindhoven and Euratom, Eindhoven, The Netherlands, Sept. 4-9, 1967.
4. Blackburn, John F.; Reethof, Gerhard; and Shearer, J. Lowen, eds.: Fluid Power Control, Technology Press of MIT, 1960.
5. Tong, L. S.: Boiling Heat Transfer and Two-Phase Flow. John Wiley and Sons, Inc., 1965.
6. Krejsa, Eugene A.; Goodykoontz, Jack H.; and Stevens, Grady H.: Frequency Response of a Forced-Flow Single-Tube Boiler. NASA TN D-4039, 1967.
7. Grace, Thomas M.; and Krejsa, Eugene A.: Analytical and Experimental Study of Boiler Instabilities Due to Feed-System - Subcooled Region Coupling. NASA TN D-3961, 1967.
8. Stenning, A. H.; and Veziroglu, T. N.: Flow Oscillations in Forced Convection Boiling. Vol. II. University of Miami (NASA CR-72122), Feb. 1967.

TABLE I. - BOILER STEADY-STATE CONDITIONS AND VALUES OF BOILER MODEL PARAMETERS

(a) English units.

Run number of reference 6	Freon flow rate, slug/sec	Vapor quality at boiler exit, percent	Subcooled length, $L_{sc}$ , ft	Boiler resistance, $R = dP_{in}/dW_t$ (lb/ft <sup>2</sup> )/(slug/sec)	Subcooled dead time, $\tau_{sc} = L_{sc}/V_{in}$ , sec	Inertance of liquid in subcooled region, $L_{sc}/A_t$ , ft <sup>-1</sup>	Change in subcooled length due to pressure change, $(\partial L_{sc}/\partial P_{L_{sc}})_{W_t}$ , (ft)/(lb/ft <sup>2</sup> )	Mass storage time constant, $R\rho_{L_{sc}}A_t(\partial L_{sc}/\partial P_{L_{sc}})_{W_t}$ , sec
Nominal water inlet temperature, 210° F								
3	0.00388	29	1.92	$1.25 \times 10^5$	0.788	3560	$2.61 \times 10^{-3}$	0.518
4	.00250	50	.84	4.25	.535	1560	.81	.547
5	.00154	63	.26	6.25	.271	480	.69	.685
Nominal water inlet temperature, 230° F								
6	0.00561	23	2.48	$0.85 \times 10^5$	0.702	4590	$1.38 \times 10^{-3}$	0.187
7	.00395	37	1.34	3.50	.542	2480	1.20	.670
8	.00250	45	.40	4.75	.255	740	.85	.640
9	.00153	47	.10	3.50	.105	185	.21	.118
Nominal water inlet temperature, 250° F								
10	0.00395	31	0.60	$4.25 \times 10^5$	0.242	1110	$1.25 \times 10^{-3}$	0.842
11	.00520	31	1.78	5.25	.546	3300	1.18	.985

(b) SI units.

Run number of reference 6	Freon flow rate, kg/sec	Vapor quality at boiler exit, percent	Subcooled length, $L_{sc}$ , M	Boiler resistance, $R = dP_{in}/dW_t$ (N/m <sup>2</sup> )/(kg/sec)	Subcooled dead time, $\tau_{sc} = L_{sc}/V_{in}$ , sec	Inertance of liquid in subcooled region, $L_{sc}/A_t$ , M <sup>-1</sup>	Change in subcooled length due to pressure change, $(\partial L_{sc}/\partial P_{L_{sc}})_{W_t}$ , m/(N/m <sup>2</sup> )	Mass storage time constant, $R\rho_{L_{sc}}A_t(\partial L_{sc}/\partial P_{L_{sc}})_{W_t}$ , sec
Nominal water inlet temperature, 372° K								
3	0.0566	29	0.58	$4.10 \times 10^5$	0.788	11 700	$16.6 \times 10^{-6}$	0.518
4	.0364	50	.26	13.9	.535	5 120	5.16	.547
5	.0224	63	.08	20.5	.271	1 570	4.40	.685
Nominal water inlet temperature, 383° K								
6	0.0818	23	0.76	$2.78 \times 10^5$	0.702	15 000	$8.80 \times 10^{-6}$	0.187
7	.0576	37	.41	11.5	.542	8 140	7.65	.670
8	.0364	45	.12	15.6	.255	2 430	5.42	.640
9	.0223	47	.04	11.5	.105	607	1.34	.118
Nominal water inlet temperature, 394° K								
10	0.0576	31	0.18	$13.9 \times 10^5$	0.242	3 640	$7.97 \times 10^{-6}$	0.842
11	.0759	31	.54	17.2	.546	10 800	7.53	.985

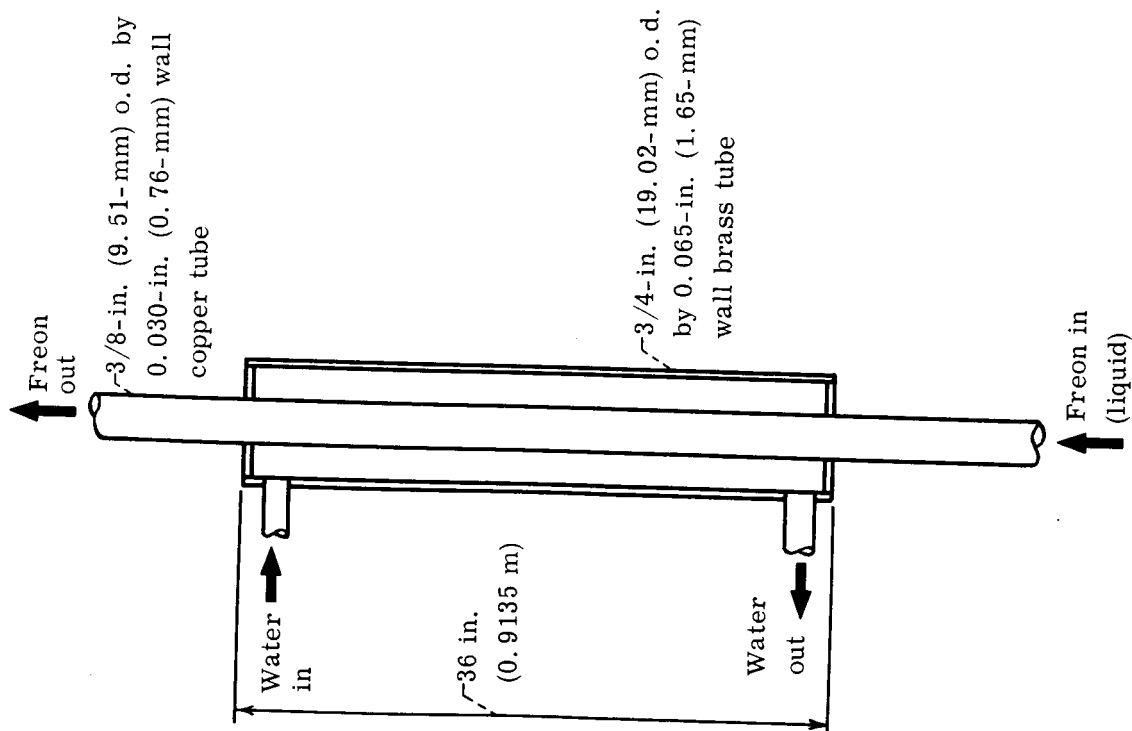


Figure 1. - Schematic of boiler used in reference 6.

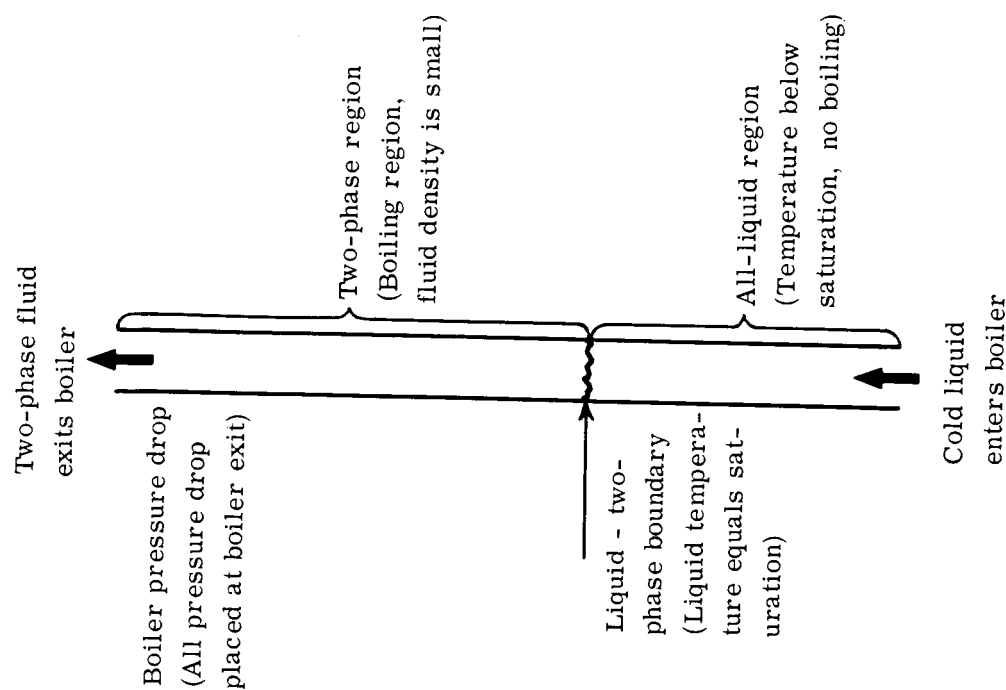


Figure 2. - Schematic of boiler indicating assumptions made in analysis.

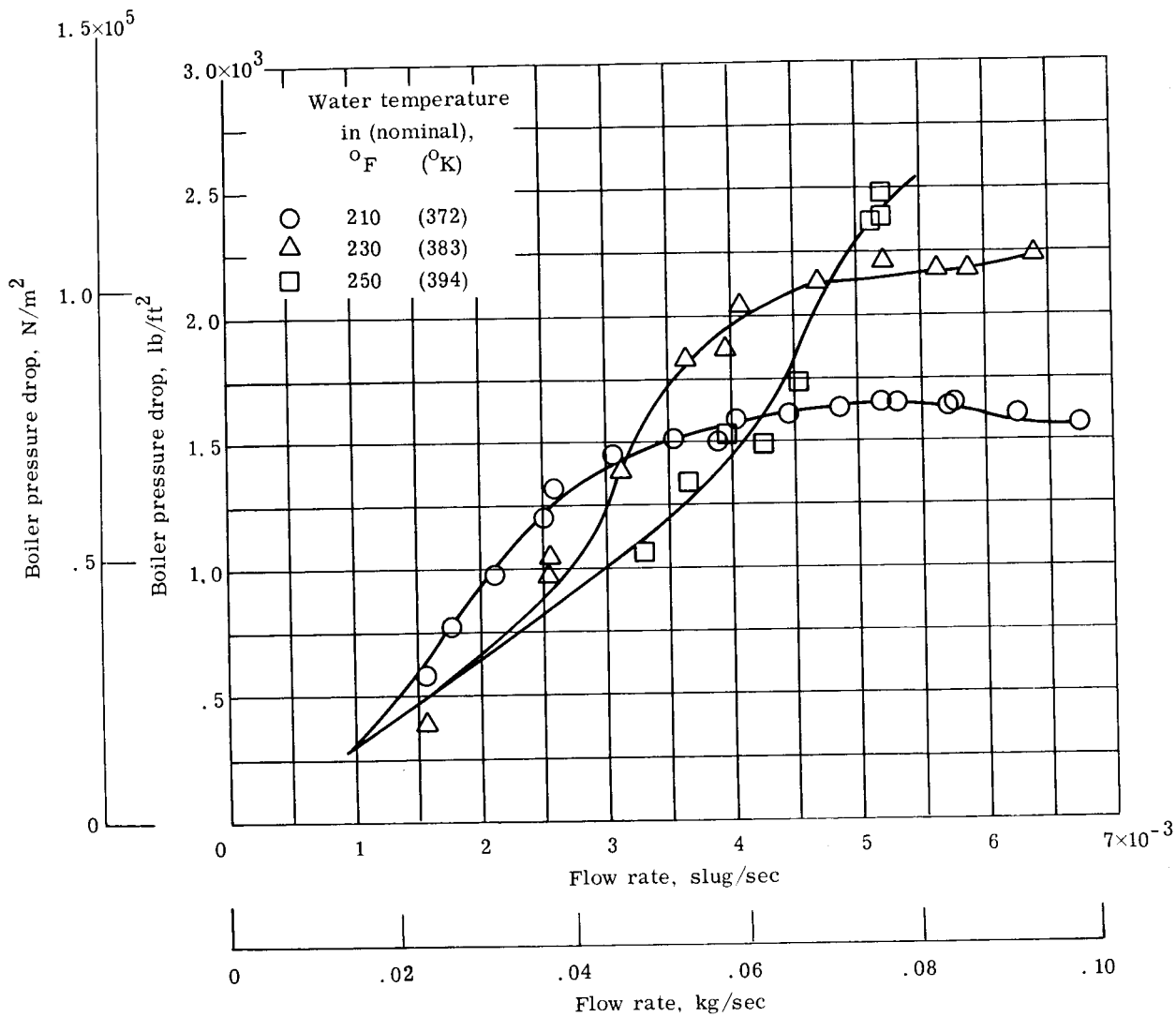


Figure 3. - Boiler pressure drop as function of flow rate for constant water inlet temperatures.



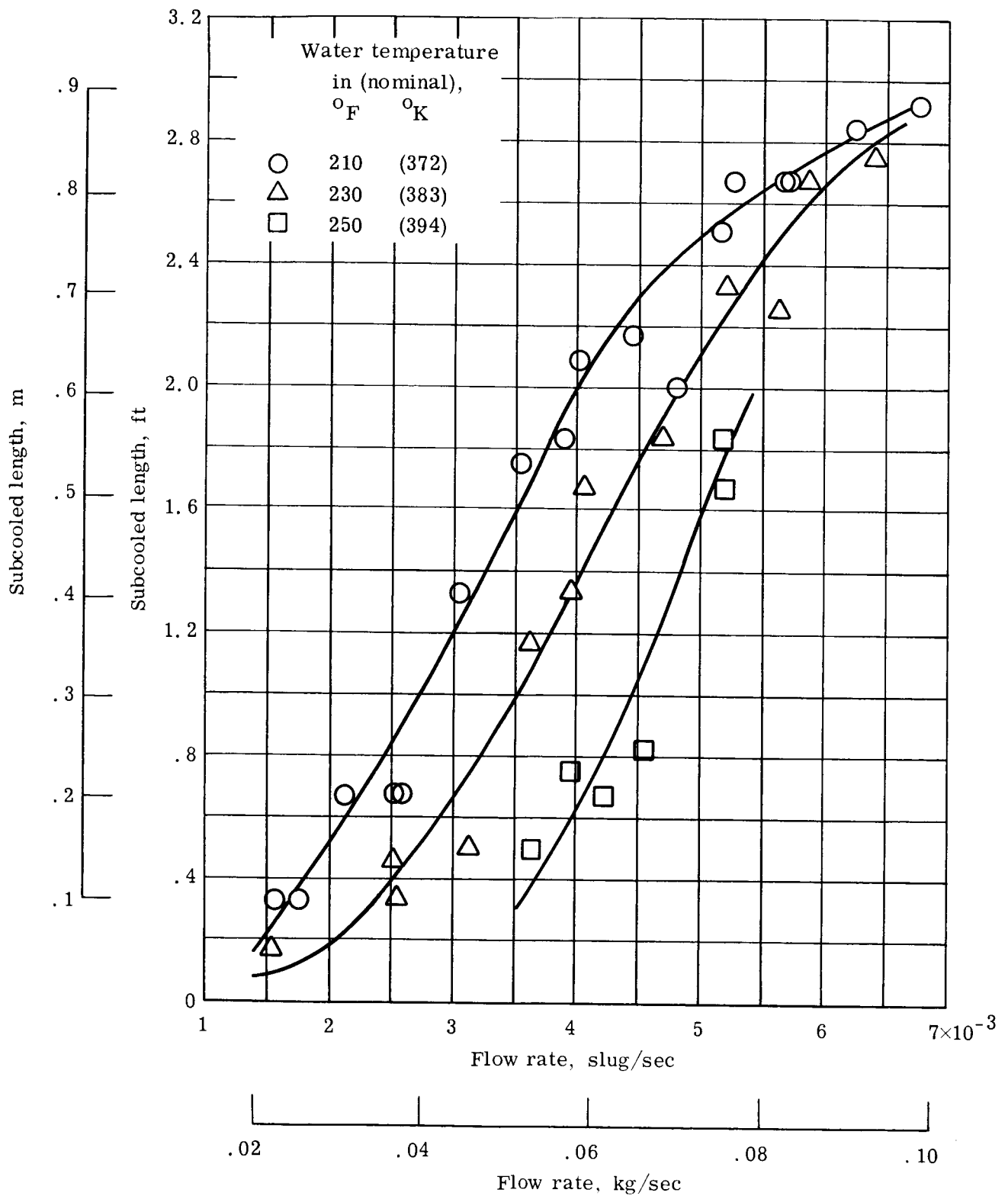
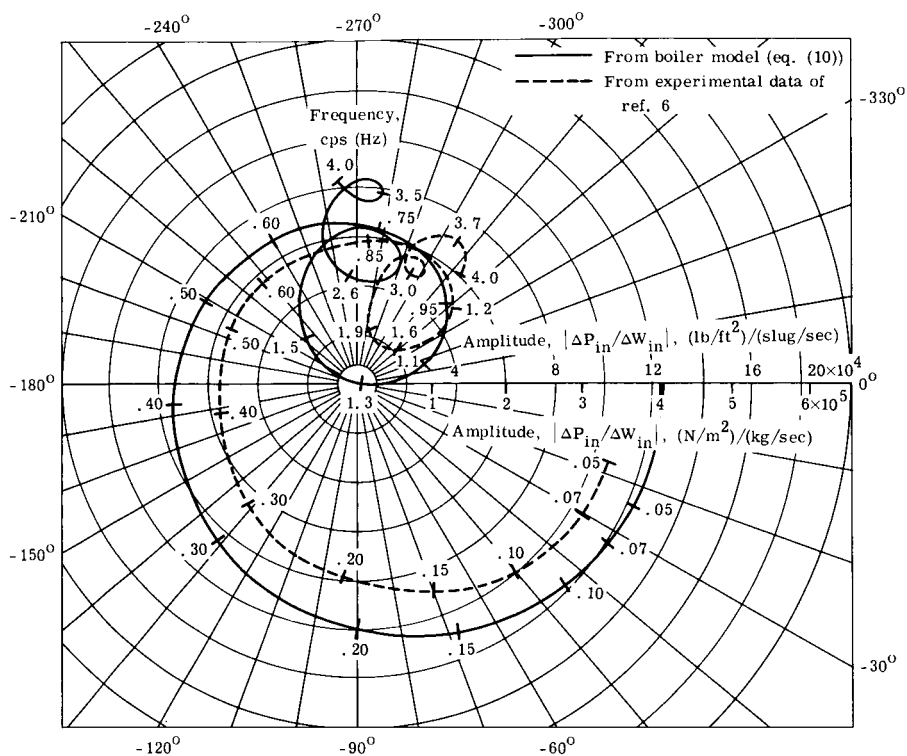
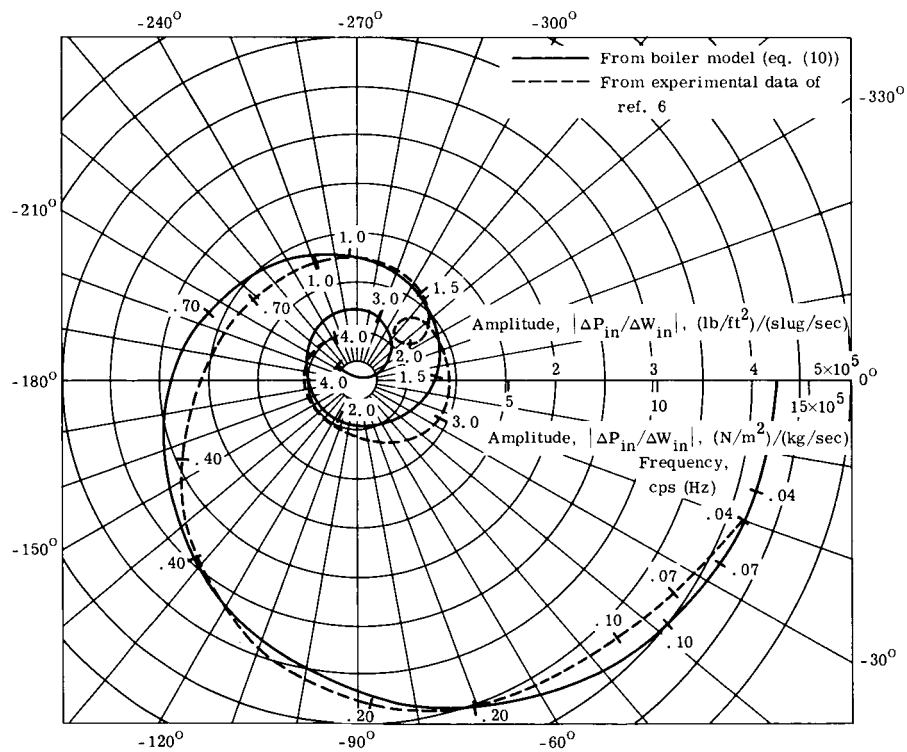


Figure 4. - Subcooled length as function of flow rate for constant water inlet temperature.

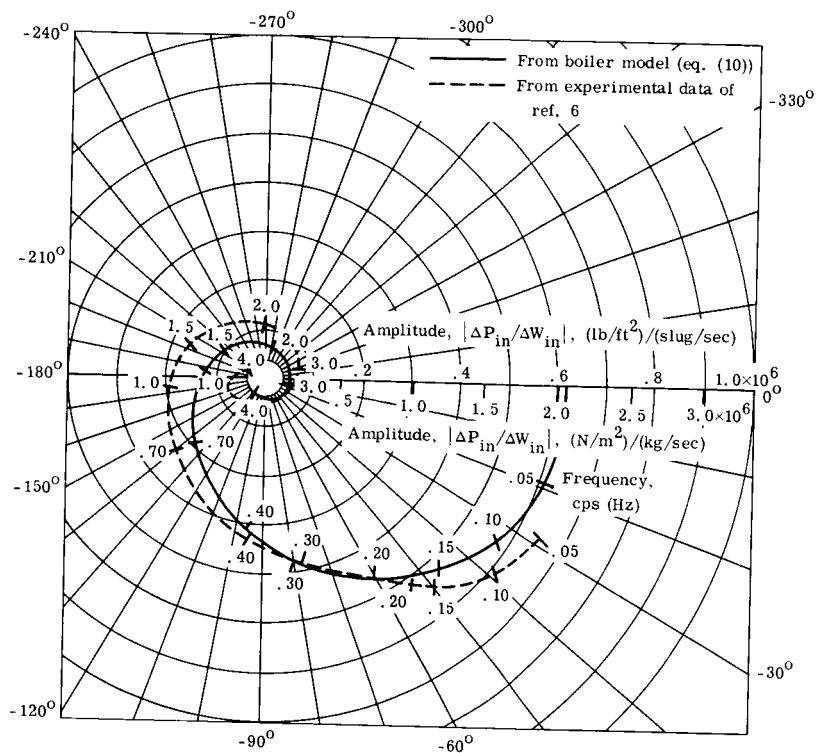


(a) Run 3.

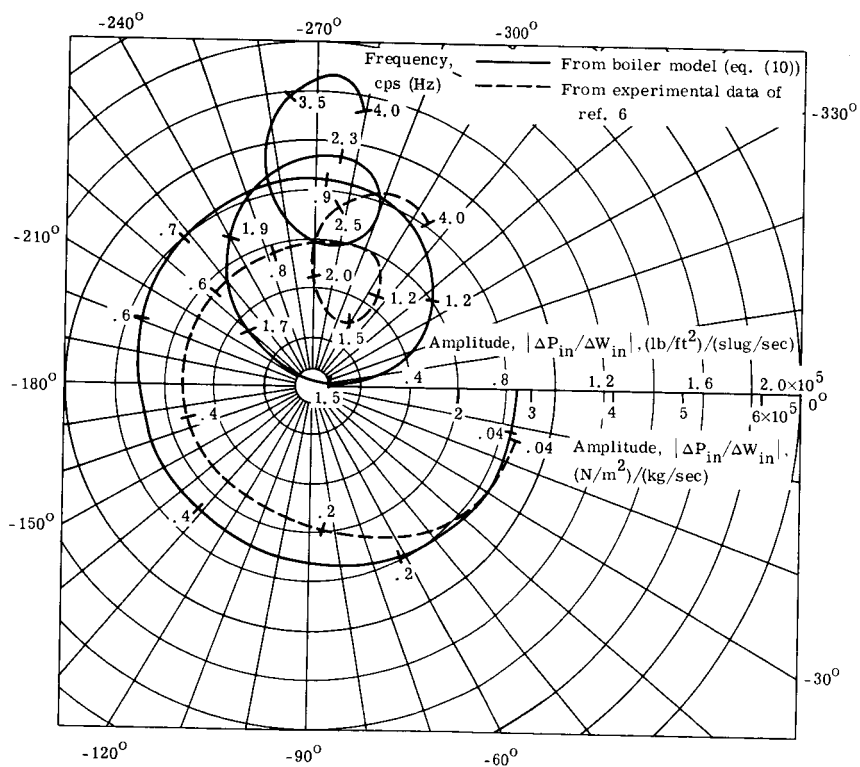


(b) Run 4.

Figure 5. - Comparison of boiler model with experimental data. (Steady-state conditions and values of model parameters listed in table I.)



(c) Run 5.



(d) Run 6.

Figure 5. - Continued.



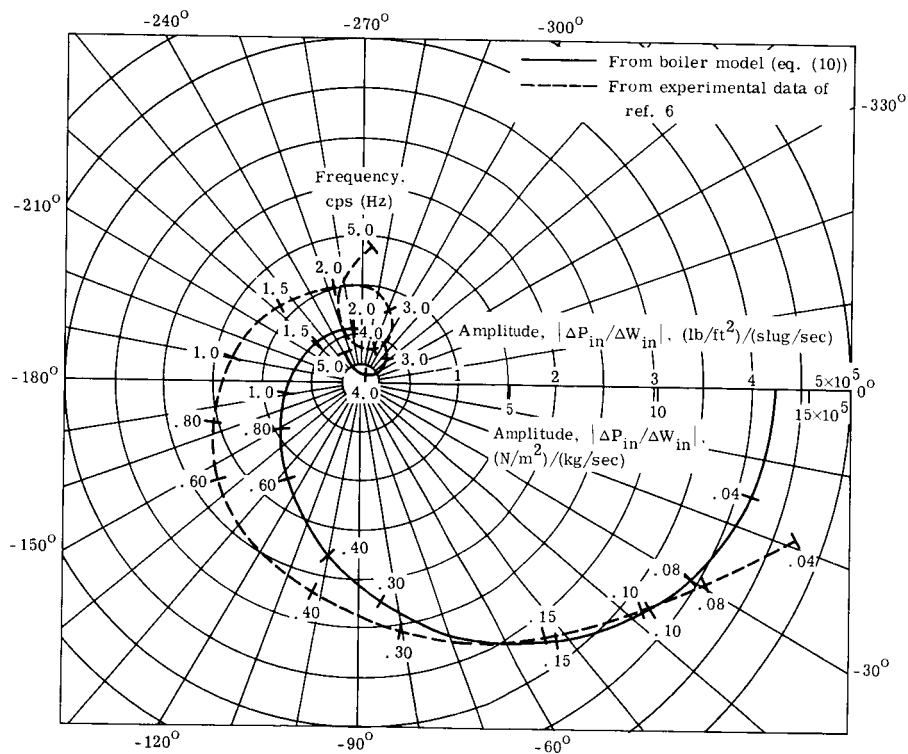
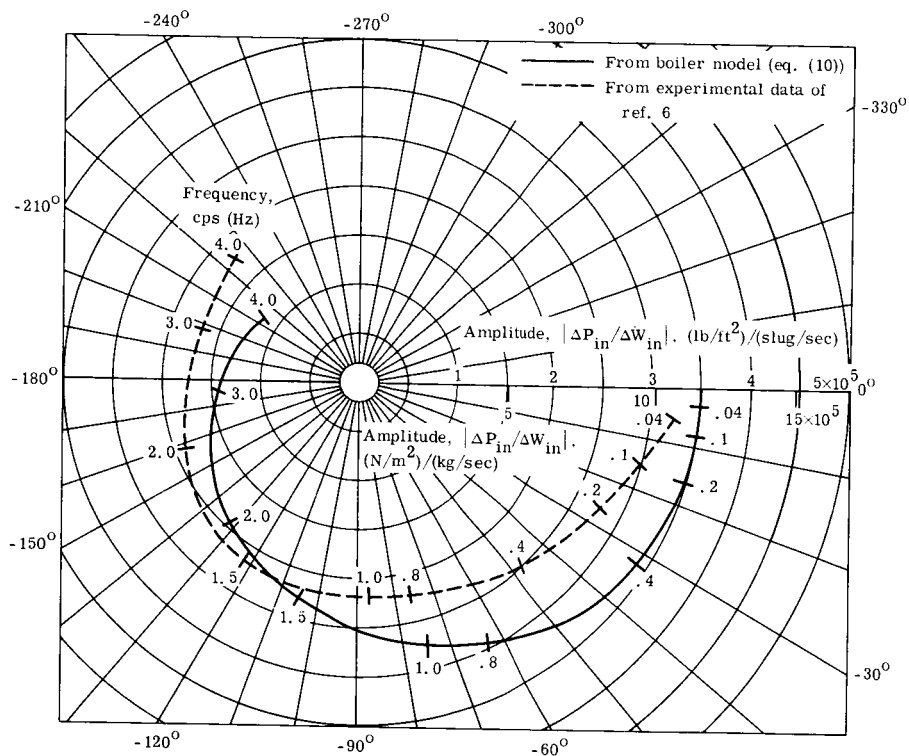


Figure 5. - Continued.

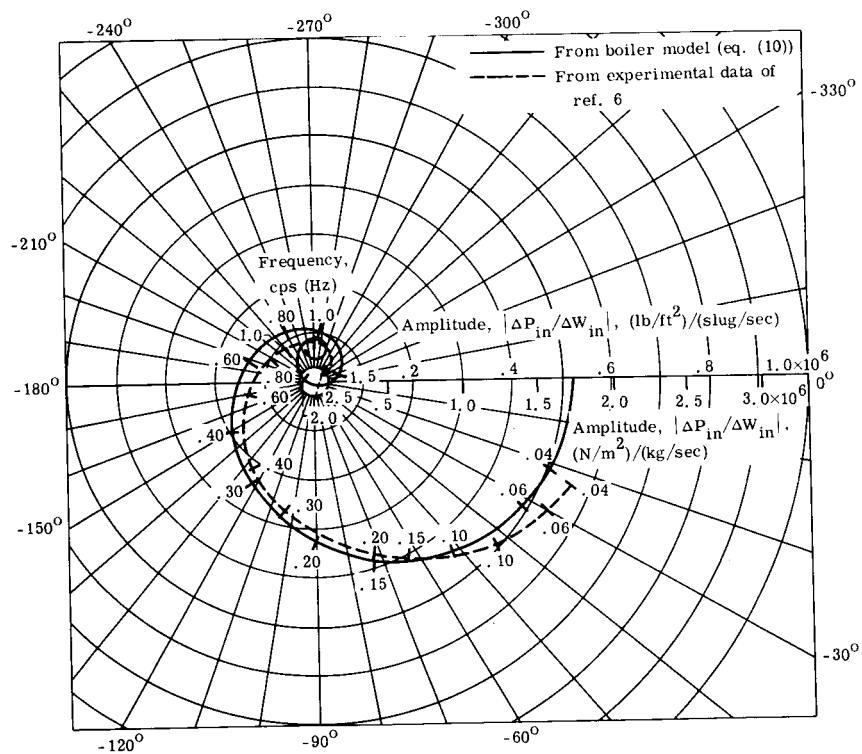
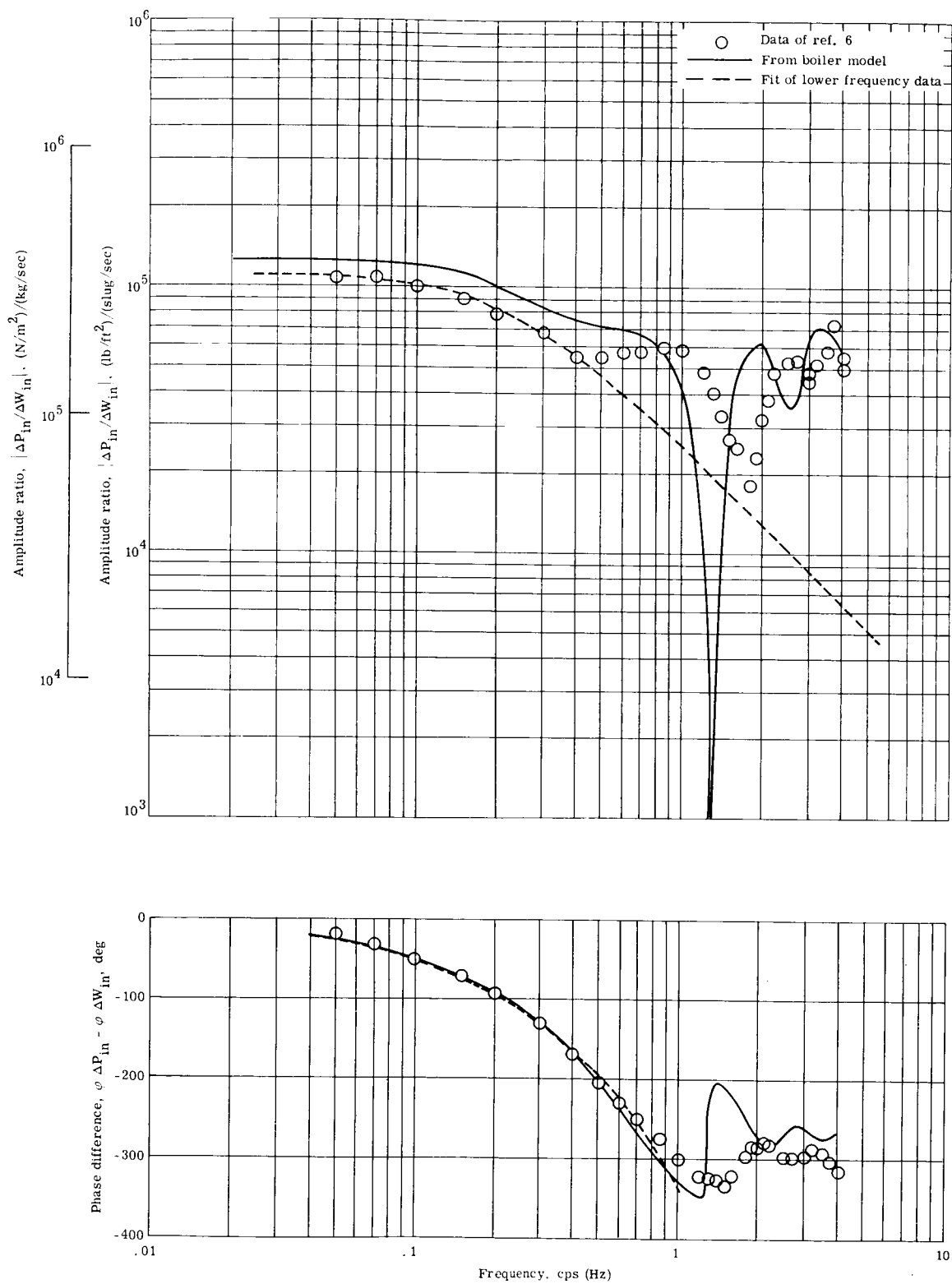
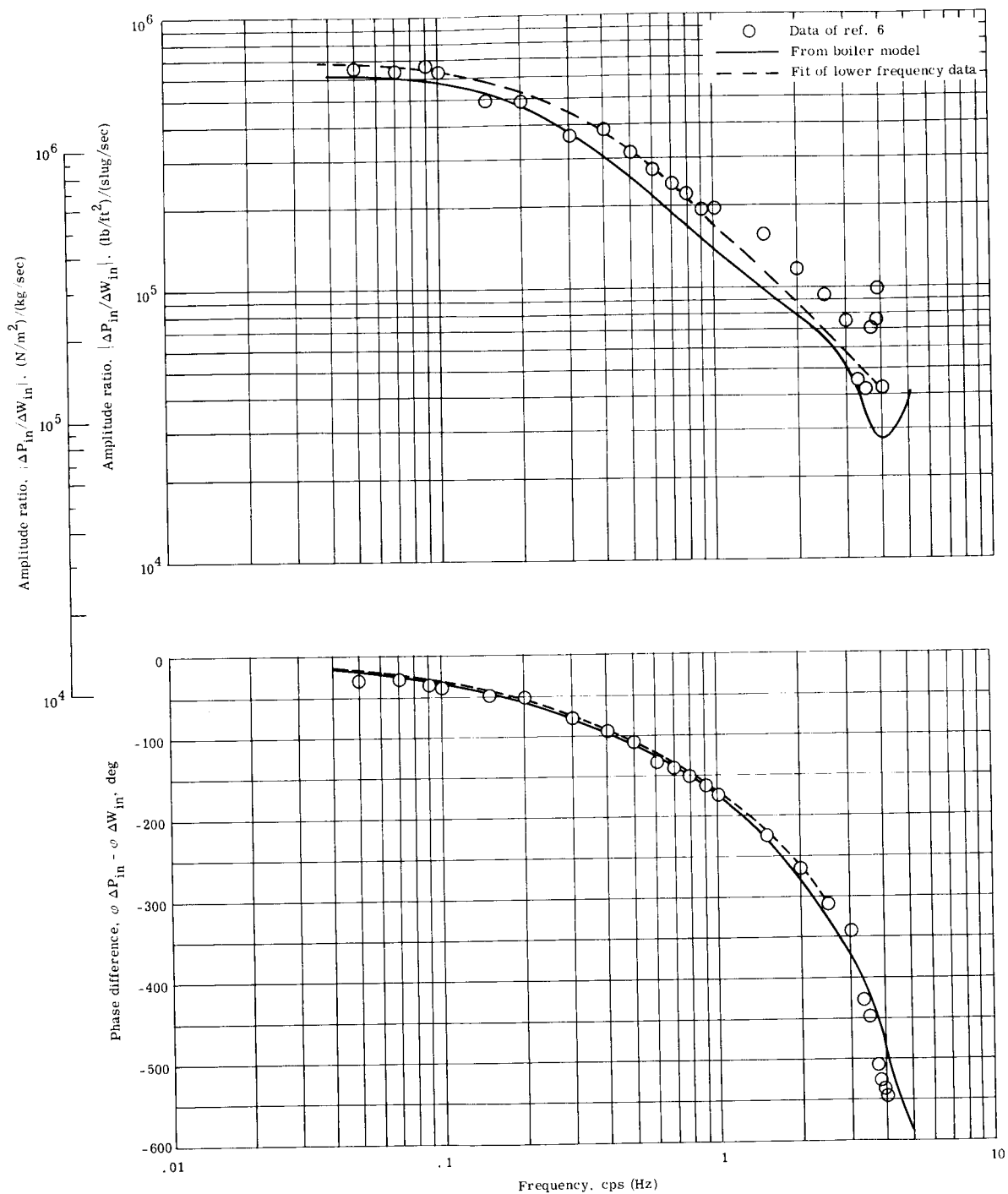


Figure 5. - Concluded.



(a) Run 3.

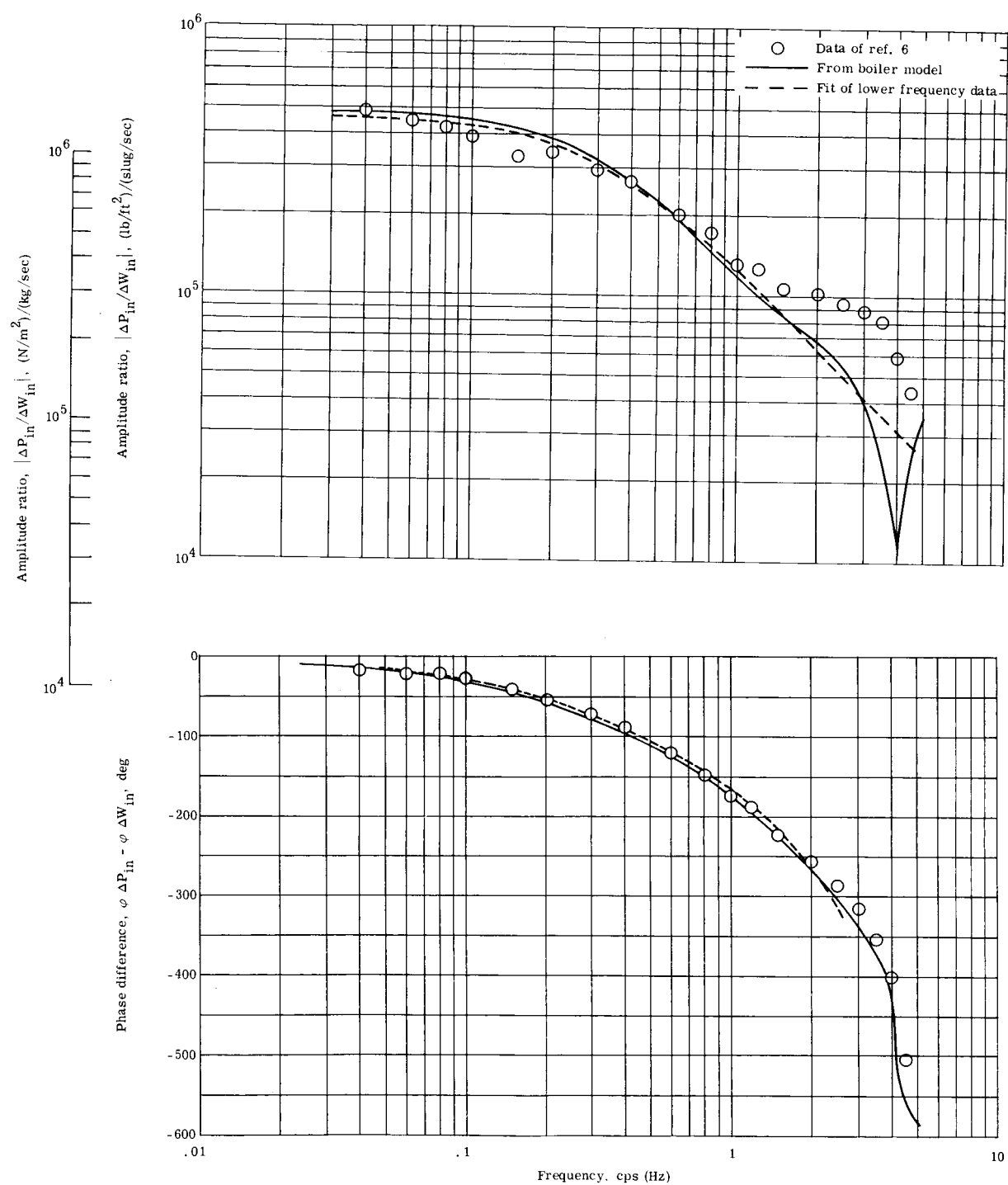
Figure 6. - Comparison of experimental data, boiler model, and data fit. (Steady-state conditions and values of model parameters listed in table I.)



(b) Run 5.

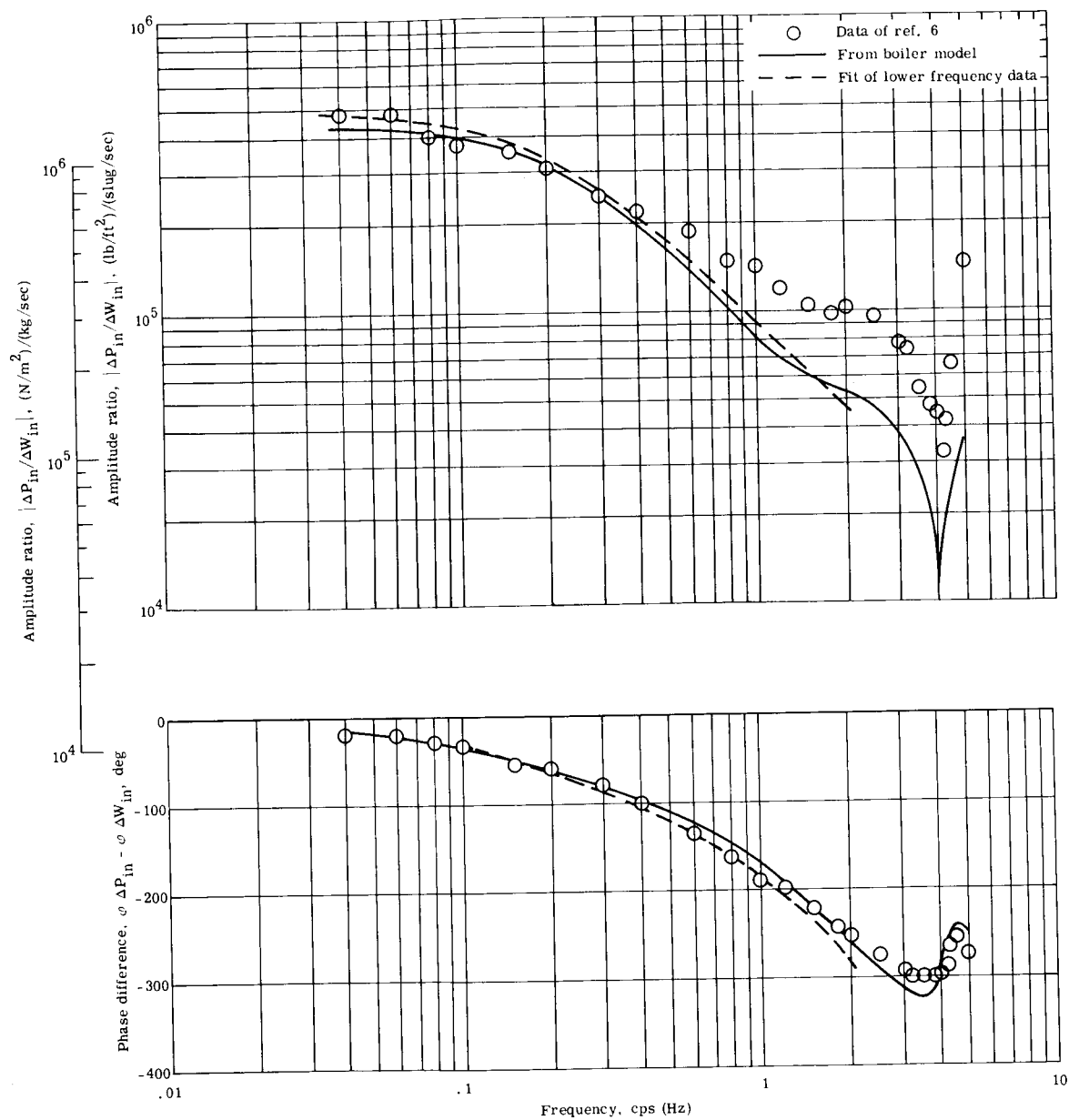
Figure 6. - Continued.





(c) Run 8.

Figure 6. - Continued.



(d) Run 10.

Figure 6. - Concluded.

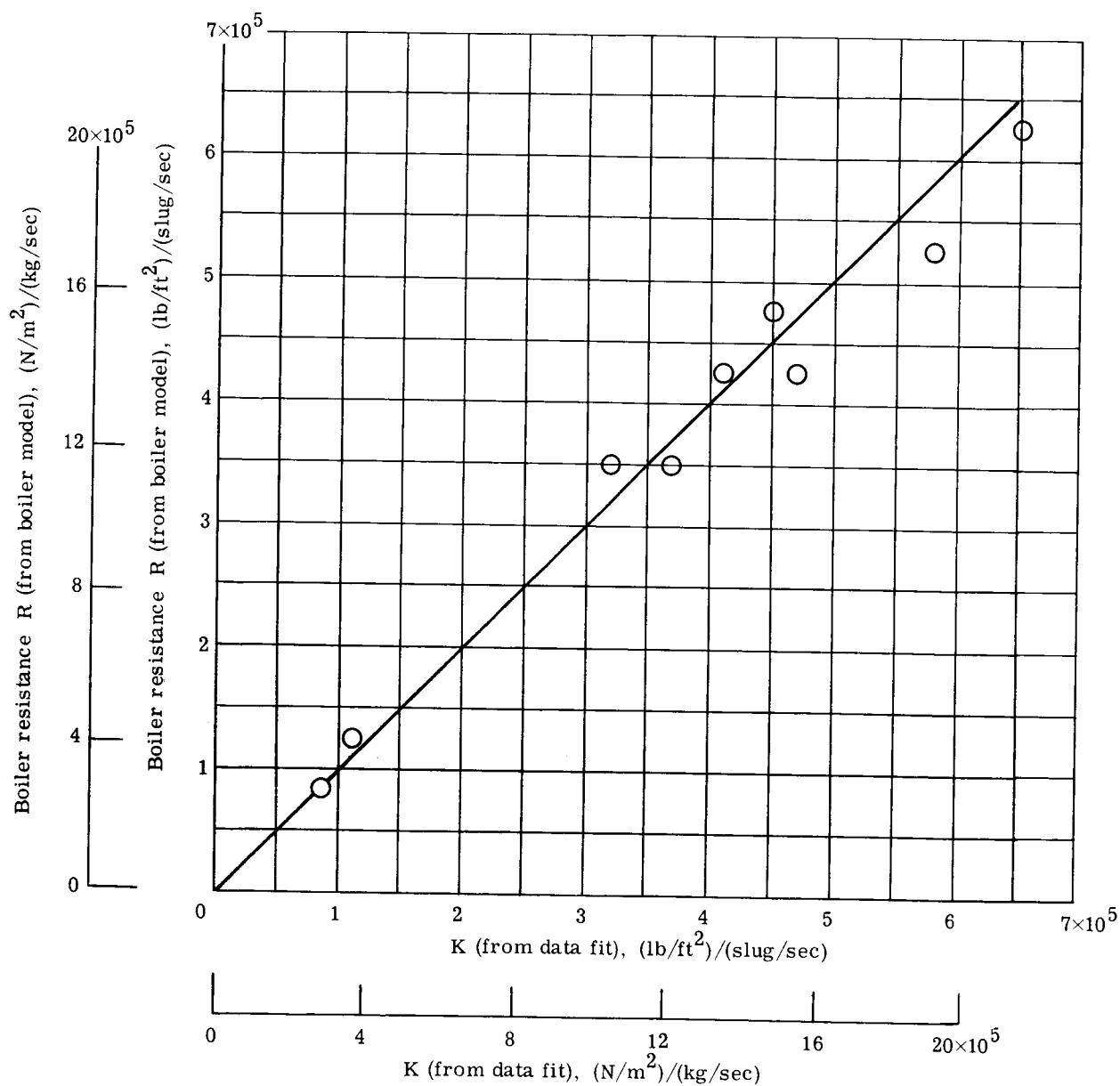


Figure 7. - Comparison of boiler resistance R (from boiler model) with K (from data fit).

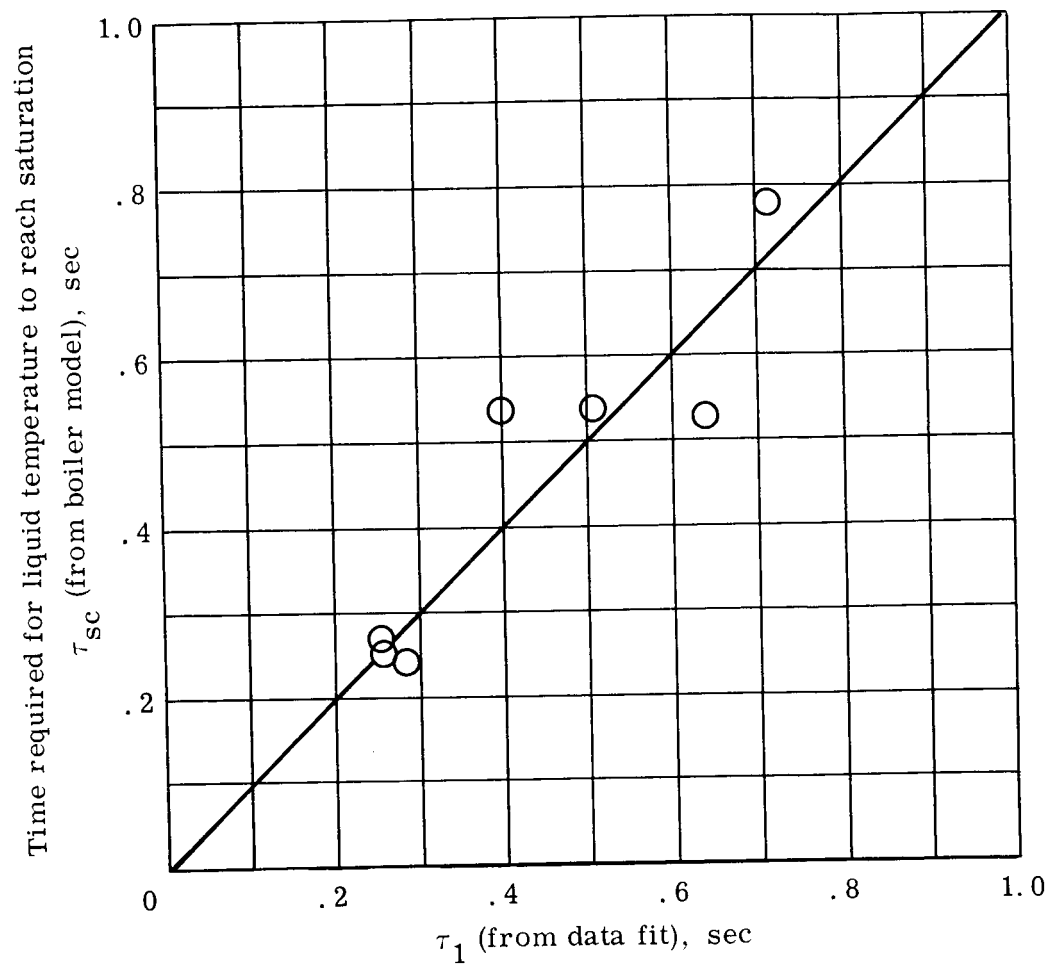


Figure 8. - Comparison of time required for liquid temperature to reach saturation  $\tau_{sc}$  (from boiler model) with  $\tau_1$  (from data fit).

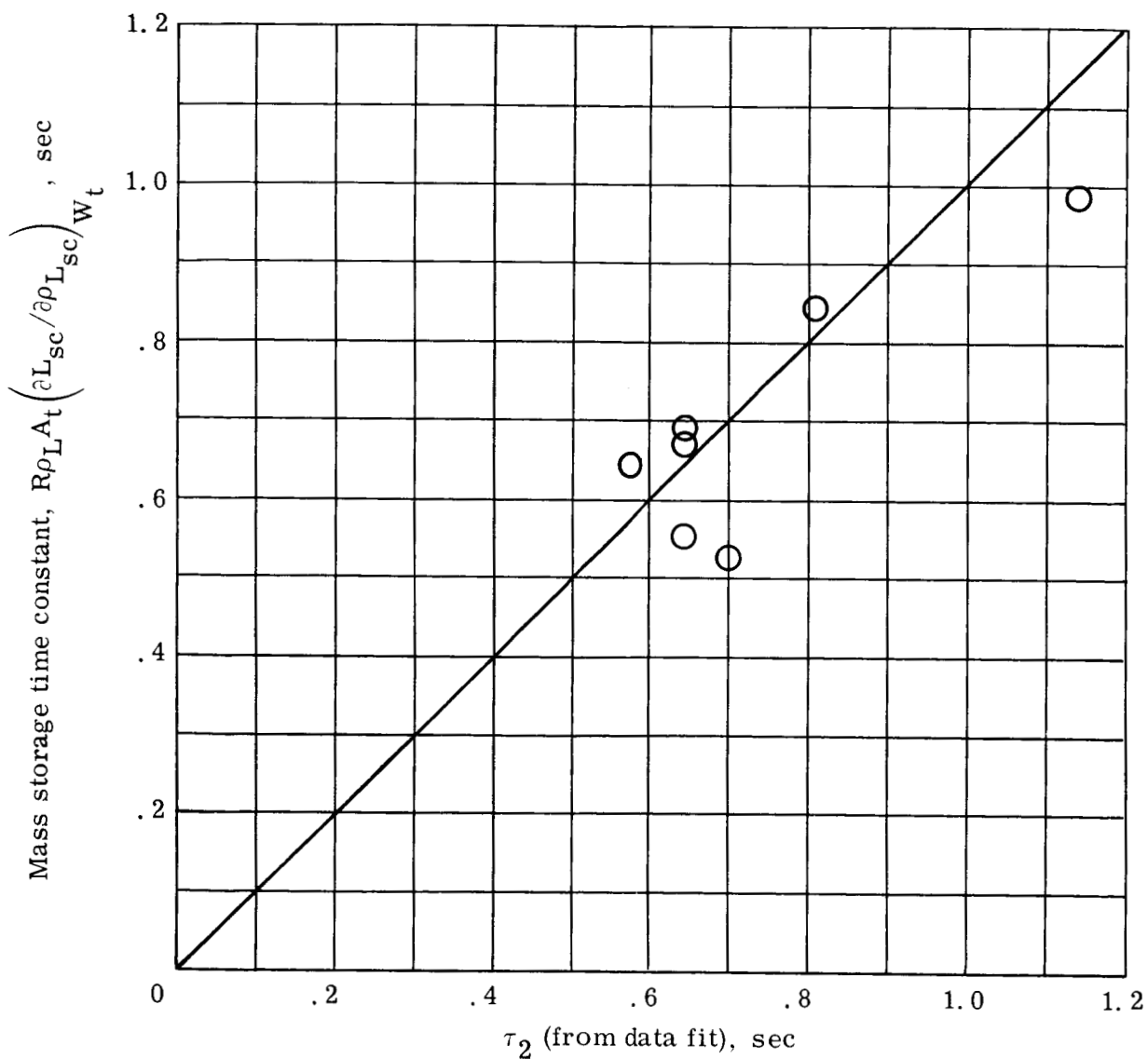


Figure 9. - Comparison of  $R\rho_L A_t \left( \frac{\partial L_{sc}}{\partial \rho_{L_{sc}}} \right) / W_t$  from boiler model with  $\tau_2$  (from data fit).